

Extending bimetric models of massive gravity to avoid to rely on the Vainshtein mechanism on local scales and the Higuchi bound on cosmological scales

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This article extends bimetric formulations of massive gravity to make the mass of the graviton to depend on its environment. This minimal extension offers a novel way to reconcile massive gravity with local tests of general relativity without invoking the Vainshtein mechanism. On cosmological scales, it is argued that the model is stable and that it circumvents the Higuchi bound, hence relaxing the constraints on the parameter space. This extension is very generic and robust and a simple specific example is described.

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Introduction. Bimetric theories of gravity attract a lot of interest, in particular with the goal of designing a theoretically consistent and observationally viable theory of massive gravitons; see e.g. Refs [1] for reviews. Those theories consider two metrics, $g_{\mu\nu}$ and $f_{\mu\nu}$, interacting through a potential that depends on $s^\alpha_\beta \equiv (\sqrt{g^{-1}f})^\alpha_\beta$ (the square-root being defined such that $s^\mu_\alpha s^\alpha_\nu = g^{\mu\alpha} f_{\alpha\nu}$).

While they offer a mathematically consistent framework for a massive graviton, one needs to rely on the Vainshtein mechanism [2], i.e. invoking the non-linear structure of the theory, in order to suppress the propagation of the extra bigravity modes in the Solar system so that they can be consistent with the test of general relativity in our local environment. On cosmological scales, the Higuchi bound [3], that is a condition between the mass of the graviton and the cosmic expansion rate, is required for the de Sitter background to be stable during inflation.

In this letter, we propose a minimal extension of these theories that allows for the mass of the massive graviton to be environmentally dependent. We shall argue that this can be implemented by extending the standard chameleon mechanism [4] and that it frees the model from the Vainshtein mechanism and the Higuchi bound, hence significantly broadening the regime of applicability of the bimetric theories.

Bimetric theories in a nutshell. They rely on the

action

$$S = \frac{M_g^2}{2} \int R[g] \sqrt{-g} d^4x + \frac{M_f^2}{2} \int R[f] \sqrt{-f} d^4x + M_g^2 m^2 \int \sum_{i=0}^4 \beta_i U_i[s] \sqrt{-g} d^4x, \quad (1)$$

where M_g and M_f are the Planck masses in the gravitational g and f sectors, respectively, and we define $\kappa \equiv M_f^2/M_g^2$. The potentials U_i ($i = 1, \dots, 4$) are defined in terms of $T_n \equiv \text{Tr}[s^n]$ as $U_0 = 1$, $U_1 = T_1$, $U_2 = \frac{1}{2} [T_1^2 - T_2]$, $U_3 = \frac{1}{6} [T_1^3 - 3T_2T_1 + 2T_3]$ and $U_4 = \frac{1}{24} [T_1^4 - 6T_1^2T_2 + 3T_2^2 + 8T_1T_3 - 6T_4]$. The cosmological constants are not included explicitly since they are already contained in U_0 and U_4 .

The cosmological analysis of this theory [5] assumes the background to be of the form

$$ds_g^2 = -dt^2 + a^2(t) \delta_{ij} dx^i dx^j, \\ ds_f^2 = \xi^2(t) [-c^2(t) dt^2 + a^2(t) \delta_{ij} dx^i dx^j]. \quad (2)$$

Among the various existing branches, Ref. [5] concluded that only the one with $H = \xi \hat{H}$ (with $H \equiv \dot{a}/a$ and $\hat{H} \equiv (a\xi)/(ac\xi^2)$) is healthy. For a matter field with energy density ρ , minimally coupled with $g_{\mu\nu}$, the Friedmann equation takes the form

$$3M_g^2 H^2 = \rho + m^2 M_g^2 R, \quad R \equiv U - \xi U_{,\xi}/4, \quad (3)$$

where $U = \beta_4 \xi^4 + 4\beta_3 \xi^3 + 6\beta_2 \xi^2 + 4\beta_1 \xi + \beta_0$. Provided that the condition [6]

$$\rho \ll M_g^2 m^2 \quad (4)$$

holds, there exists a stable cosmological background whose Friedmann equation behaves as in the standard cosmology, $H^2 \propto \rho$ up to corrections that are negligible below a certain high energy scale. The cosmological

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evolution then deviates from the standard one at high energy, $\rho \gtrsim M_g^2 m^2$.

It was also shown [5] that the massive tensor mode has a mass

$$m_T^2 = \frac{1 + \kappa \xi^2}{\kappa \xi^2} m^2 \Gamma, \quad \Gamma \equiv \xi J + (c - 1) \xi^2 J_{,\xi} / 2, \quad (5)$$

where $J = R_{,\xi} / 3$. Given the condition (4), the mass can be made light enough ($m_T^2 \ll m^2$) so as to give a new phenomenology detectable [6] by gravitational wave detectors only if the parameters of the theory are fine-tuned so that $\Gamma \approx 0$.

This latter condition is also required for the implementation of the Vainshtein mechanism (so as to suppress any fifth force arising from the extra gravity degrees of freedom). This mechanism may work only if

$$\left| \frac{d \ln \Gamma}{d \ln \xi} \right| \gg 1, \quad (6)$$

which generically holds for $\Gamma \approx 0$. To finish, the stability of de Sitter backgrounds also requires that $m_T^2 > 2H^2$, referred to as the Higuchi bound [3].

Allowing for a scaling of the interaction potential. Suppose the coefficients of the potential depends on the environment in such a way that they scale as (we shall discuss how to implement this idea below) $\beta_i \propto \rho$, then it is clear that $U \propto \rho$ and $\Gamma \propto \rho$ so that $m_T^2 \propto \rho$. We then have

$$\frac{m_T^{\text{loc}}}{m_T^{\text{cosm}}} \approx \sqrt{\frac{\rho_{\text{loc}}}{\rho_{\text{cosm}}}} \sim 10^5$$

if we assume that the mean density on Solar system scales is given by the dark matter density $\rho_{\text{loc}} \sim \rho_{\text{DM, SolarSyst.}} \simeq 1.4 \times 10^{-19} \text{ g.cm}^{-3}$ and that the cosmological density is $\rho_{\text{cosm}} = 3H_0^2 \Omega_{m0} / 8\pi G \sim 10^{-29} \text{ g.cm}^{-3}$.

The requirement that the massive graviton is heavy enough on Solar system scales imposes $m_T^{\text{loc}} \gg 1 \text{ AU}^{-1} \approx 8 \times 10^{-18} \text{ eV}$. Combined with the assumed scaling, this sets the bound at cosmological scales as $m_T^{\text{cosm}} \gg 8 \times 10^{-23} \text{ eV}$. In particular, this constraint allows cosmological values as light as $m_T^{\text{cosm}} \simeq 1000 \text{ pc}^{-1} = 4 \times 10^{-20} \text{ eV}$, which would not be possible otherwise. With scaling of the form $m_T \propto \sqrt{\rho}$, the bound on the cosmological value of the mass of the massive tensor mode coming from Solar system constraints is relaxed and leaves a larger space for non-trivial cosmological phenomenology.

As we shall see later, such a scaling may also alleviate the constraint required for the stability of the cosmological evolution all the way up to the scale at which the effective theory describing bigravity breaks down. The constraint due to this requirement would be the most stringent in the standard bigravity. This cosmological stability condition is often called the generalized Higuchi

condition ¹,

$$m_T^2 > \mathcal{O}(1) \times H^2, \quad (7)$$

where m_T and H are the mass of the massive tensor mode and the Hubble expansion rate at the time of interest. If a mechanism allows for the scaling $m_T \propto \sqrt{\rho}$, then the generalized Higuchi condition is almost independent of the scale and thus will be automatically satisfied at all scales once it is satisfied at one scale.

Implementing the mechanism. The simplest way to get an environmental dependence of the coefficients of the potentials U_i is to promote them to functions of a scalar field ² and to implement the chameleon mechanism [4]. This is easily achieved by extending the action (1) by first, letting β_i depend on a scalar field ϕ as

$$\beta_i = \beta_i(\phi), \quad (8)$$

then allowing for a non-minimal coupling of the matter sector

$$S_{\text{mat}} = \int \mathcal{L}_{\text{mat}}(\psi, \tilde{g}_{\mu\nu}) \sqrt{-\tilde{g}} d^4x, \quad (9)$$

where $\tilde{g}_{\mu\nu} = A^2(\phi) g_{\mu\nu}$ and $A(\phi)$ is a coupling function, assumed to be universal, and by adding a kinetic term of the scalar field

$$S_{\text{kin}} = -\frac{1}{2} \int g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \sqrt{-g} d^4x. \quad (10)$$

This defines an extended bigravity theory in the Einstein-frame. The Jordan-frame description of the same theory can be defined by shifting to $\tilde{g}_{\mu\nu}$ and $\tilde{f}_{\mu\nu} = A^2(\phi) f_{\mu\nu}$ that let s invariant. The interaction terms $M_g^2 m^2 \sum_i \beta_i(\phi) U_i$ play the role of a potential for ϕ as well. Finally, we define the coupling strength of the field to matter by $\alpha(\phi) \equiv d \ln A / d \phi$. No potential was added to (10) since it can be absorbed in a redefinition of $\beta_0(\phi)$.

The dynamics of such a theory is governed by two Einstein equations, a Klein-Gordon equation and a conser-

¹ The bound derived in [7] is for homogeneous and isotropic perturbation and thus is the condition for the avoidance of an IR ghost, which a priori is not necessarily the right condition for theoretical consistency [8]. The no-ghost condition for general inhomogeneous and anisotropic perturbation in the high k limit was later derived in [5].

² In the context of theories of single massive graviton, the idea of promoting the coefficients of the graviton interaction terms to functions of a scalar field is not new. For example, see [9].

vation equation, which take the form

$$\begin{aligned}
G_{\mu\nu}[g] &= \frac{1}{M_g^2} \left[T_{\mu\nu} + \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} \partial_\alpha \phi \partial_\beta \phi g^{\alpha\beta} g_{\mu\nu} \right] \\
&\quad + m^2 \sum_{i=0}^3 (-1)^{i+1} \beta_i(\phi) Y_{\mu\nu}^{(i)}, \\
G_{\mu\nu}[f] &= m^2 \sum_{i=0}^3 (-1)^{i+1} \beta_{4-i}(\phi) \hat{Y}_{\mu\nu}^{(i)}, \\
\Box \phi &= -\alpha T_{\mu\nu} g^{\mu\nu} - M_g^2 m^2 \sum_{i=1}^3 \beta'_i(\phi) U_i[s], \\
\nabla_\mu T^{\mu\nu} &= \alpha(\phi) T^{\mu\nu} \partial_\mu \phi.
\end{aligned} \tag{11}$$

where

$$\begin{aligned}
Y_{\mu\nu}^{(0)} &= g_{\mu\nu}, \quad Y_{\mu\nu}^{(1)} = [s_\mu^\alpha - T_1 \delta_\mu^\alpha] g_{\nu\alpha}, \\
Y_{\mu\nu}^{(2)} &= g_{\mu\alpha} s_\beta^\alpha s_\nu^\beta - T_1 g_{\mu\alpha} s_\nu^\alpha + \frac{1}{2} [T_1^2 - T_2] g_{\mu\nu}, \\
Y_{\mu\nu}^{(3)} &= g_{\mu\alpha} s_\beta^\alpha s_\gamma^\beta s_\nu^\gamma - T_1 g_{\mu\alpha} s_\beta^\alpha s_\nu^\beta + \frac{1}{2} [T_1^2 - T_2] g_{\mu\alpha} s_\nu^\alpha \\
&\quad - \frac{1}{6} [T_1^3 - 3T_2 T_1 + 2T_3] g_{\mu\nu},
\end{aligned} \tag{12}$$

and $\hat{Y}_{\mu\nu}^{(i)}$ is defined in the same way as $Y_{\mu\nu}^{(i)}$ with $g_{\mu\nu} \leftrightarrow f_{\mu\nu}$.

On *cosmological scales*, adopting the ansatz (2), the field equations reduce to

$$3H^2 = \frac{1}{M_g^2} \left[\rho A^4 + \frac{1}{2} \dot{\phi}^2 \right] + m^2 R, \tag{13}$$

$$3\hat{H}^2 = \frac{1}{4\kappa} U_{,\xi}, \tag{14}$$

$$2\dot{H} = -\frac{1}{M_g^2} \left[(\rho + P) A^4 + \dot{\phi}^2 \right] + m^2 \xi (c-1) J, \tag{15}$$

$$2\dot{\hat{H}} = m^2 \frac{1-c}{\kappa} \xi J, \tag{16}$$

and

$$\ddot{\phi} + 3H\dot{\phi} = -\alpha A^4 (\rho - 3P) + M_g^2 m^2 Q_{,\phi}, \tag{17}$$

with $Q(\xi, \phi) \equiv 4R - 3(c-1)\xi J$. Note that ρ is the energy density in the Jordan frame, defined by $T_{\mu\nu}(\partial/\partial t)^\mu(\partial/\partial t)^\nu \equiv \rho A^4$. Hence the conservation equation implies that $\rho \propto (Aa)^{-3}$ for pressureless matter, for example. The functions U , R and J , defined as before, now depend on both ξ and ϕ . By combining Eqs. (13) and (15) with Eq. (17), it is straightforward to derive the constraint

$$\left[\dot{\xi} + (1-c)H\xi \right] \sum_{i=0}^2 \binom{2}{i} \beta_{i+1} \xi^i = \frac{c\xi}{3} \sum_{i=0}^3 \binom{3}{i} \dot{\beta}_{i+1} \xi^i \tag{18}$$

On *local scales*, we assume that the spacetime is close to Minkowski and solve, in the chameleon spirit [4], the Klein-Gordon equation

$$\frac{d^2 \phi}{dr^2} + \frac{2}{r} \frac{d\phi}{dr} = \rho \alpha A^4 - M_g^2 m^2 Q_{,\phi}, \tag{19}$$

assuming that the local density is $\rho = \rho_{\text{loc}}$ and $\xi = \xi_{\text{loc}}$.

Mechanism. For simplicity, and as a proof of concept, we assume that

$$\beta_i(\phi) = -c_i f(\phi), \tag{20}$$

so that $U[\beta_i(\phi), \xi] \rightarrow -U[c_i, \xi] f(\phi)$ since it is a linear function of the coefficients β_i . The same applies to the functions R , J and Q . The universal coefficient function $f(\phi)$ is chosen as a decreasing function of ϕ and the coupling function is assumed to be given by

$$A = e^{\beta \phi / M_g}, \tag{21}$$

so that $\alpha = \beta / M_g$ is a constant. Now, in the local region, $\xi = \xi_{\text{loc}}$ is constant and $c = 1$. On cosmological scales it can be shown [5] that $\xi \rightarrow \xi_c$ constant and $c \rightarrow 1$. We can thus argue that $\xi_{\text{loc}} = \xi_c$ and $c = 1$ at late time and on cosmological scales (denoted by infinity).

The potential appearing in the Klein-Gordon equations (17) and (19) has a minimum at a value $\phi = \phi_*(\rho, \xi, c)$, which will mostly be a function of ρ under the above conditions. Hence, $m_T^2 \propto f(\phi)$ is essentially a function of ρ . This clearly realizes the idea discussed above. The exact scaling of ϕ_* and thus m_T^2 with ρ depends on the form of the function $f(\phi)$ so that we need to take more specific examples to put numbers. Nonetheless this discussion at the very least demonstrates the generality of the mechanism. It can be easily extended by allowing for each β_i to have a different scaling.

Example. Let us further assume the form

$$f = e^{-\phi/M} = e^{-\lambda \phi / M_g}, \tag{22}$$

where M is a mass scale and $\lambda = M_g/M$. Clearly, the local and cosmological values of the field are related by

$$(\lambda + 4\beta)(\phi_\infty - \phi_{\text{loc}}) = M_g \ln \frac{\rho_{\text{loc}}}{\rho_\infty} \tag{23}$$

as long as a backreaction is small and assuming the field has settled at the minimum of its potential. We conclude that the masses of the massive tensor mode at local and cosmological scales are related by

$$\frac{m_{T,\text{loc}}^2}{m_{T,\infty}^2} = \left(\frac{\rho_{\text{loc}}}{\rho_\infty} \right)^{\frac{\lambda}{\lambda+4\beta}}. \tag{24}$$

Assuming $\beta = \mathcal{O}(1)$ and $M \ll M_g$ (so that $\beta \ll \lambda$), we end up with the announced scaling $m_T^2 \propto \rho$. The situation is then similar as in the standard chameleon mechanisms [4].

5th force. While the fifth force associated with the gravity degrees of freedom are suppressed by our mechanism, the scalar field ϕ is also responsible for such a force. Equation (19) is similar to the standard chameleon case [4] but with a potential $V = -M_g^2 m^2 Q(\xi, \phi) =$

$-M_g^2 m^2 Q(\xi, 0) e^{-\lambda\phi/M_g}$. Around a massive body of density ρ_c and radius R_c , the field will have a profile interpolating from ϕ_c inside to ϕ_{loc} outside, these two values being related by the same relation as Eq. (23). For the mechanism to apply, we need to have $Q(\xi, 0) < 0$, which can be achieved if $c_i > 0$ and $c \simeq 1$ for example. Asymptotically, the mass of the scalar field is given by the second derivative of the effective potential at its minimum. In the limit $\beta \ll \lambda$ it is well approximated by $m_\phi^2 \sim \lambda\beta\rho/M_g^2$. The field becomes massive in high density environment and the theory can pass tests on fifth force. The amplitude of the fifth force depends on the density and size of the objects and environment as well as the parameter of the model, and in particular if the thin shell approximation holds. Indeed, depending on the parameters, violations may be detected in space. We shall detail the full analysis of the parameter space in a companion paper [10].

Stability on de Sitter. Since the general covariance of the theory is not broken, it is expected that one massless graviton should still exist. Nonetheless, it is important to know what is the mass of the other massive graviton, and how it depends on the value of the scalar field, and, thus on the environment. We emphasize that the massless mode does not need to coincide with the tensor mode of the physical metric. Finally, we also want to know whether the final state of the universe would be stable. To address these issues, we assume the universe is asymptotically well-described by a de Sitter background with $\phi = \text{const}$, and thus suppose that the matter enjoys the equation of state $P = -\rho$. As in the standard bigravity, two different branches exist since for $\dot{\phi} = 0$ the constraint equation (18) reduces to $(1 - c)(\beta_3\xi^2 + 2\beta_2\xi + \beta_1) = 0$. We shall focus on the normal branch, i.e. the one which does not suffer from strong-coupling and/or ghosts. It is defined by the condition $c = 1$. For the example (22), the Friedmann equations give

$$H^2 = \frac{m^2}{3} e^{-\frac{\lambda\phi}{M_g}} \sum_{i=0}^3 \binom{3}{i} c_i \xi^i + \frac{\rho}{3M_g^2} e^{\frac{4\beta\phi}{M_g}}, \quad (25)$$

$$H^2 = \frac{m^2}{3\kappa\xi} e^{-\frac{\lambda\phi}{M_g}} \sum_{i=0}^3 \binom{3}{i} c_{i+1} \xi^i, \quad (26)$$

where both ξ and ϕ have constant values. After using Eq. (17) to eliminate ρ , it implies

$$\beta \sum_{i=0}^3 \binom{3}{i} \xi^i \left(c_i \xi - \frac{c_{i+1}}{\kappa} \right) + \frac{\lambda\xi}{4} \sum_{i=0}^4 \binom{4}{i} c_i \xi^i = 0. \quad (27)$$

This algebraic constraint can be solved to get ξ as a function of $(c_i, \lambda, \beta, \kappa)$. The solutions depend neither on ϕ nor on ρ , but only on the parameters of the theory. In other words, they do not depend on the environment. In particular, if $\beta = \mathcal{O}(1)$, $M \ll M_g$ and $M \ll M_f^2/M_g$ (so that $\beta \ll \lambda$ and $\beta\kappa^{-1} \ll \lambda$) and if $c_i = \mathcal{O}(1)$, then Eq.

(27) leads to

$$\sum_{i=0}^4 \binom{4}{i} c_i \xi^i \approx 0, \quad (28)$$

unless fine-tuned. It then follows that ξ is a function of the c_i alone.

Let us now study the stability of this background against cosmological perturbations. Both tensor modes (with two polarizations each) propagate with the speed of light (for high k 's). However, one can show that the eigen-mass spectrum contains one eigen-state with a vanishing squared-mass eigenvalue (i.e. one of the gravitons remains massless), whereas the other one satisfies

$$\frac{m_T^2}{H^2} = \frac{3(1 + \kappa\xi^2)(c_3\xi^2 + 2c_2\xi + c_1)}{c_4\xi^3 + 3c_3\xi^2 + 3c_2\xi + c_1}. \quad (29)$$

This explicitly shows that m_T^2/H^2 , being only a function of ξ , is independent of the environment. This leads to the crucial fact that, the larger H the larger the mass of the graviton. We also found that the no-ghost condition for the vector modes requires that $m_T^2 > 0$, whereas their speed of propagation (for high k 's) turns out to be unity (as expected from this de Sitter-invariant background). Finally the scalar modes can be shown to have two modes propagating with speed of light (for high k 's), which are both not ghosts if the following relation holds

$$\frac{m_T^2}{H^2} > 1 + \sqrt{1 + 6\lambda^2\kappa\xi^2(1 + \kappa\xi^2)}. \quad (30)$$

For $\lambda \neq 0$, this is a relation stronger than the usual Higuchi bound,

$$m_T^2 > 2H^2, \quad (\lambda = 0). \quad (31)$$

Because of the condition (29), the no-ghost condition (30) is also independent of the environment.

Discussion. This letter describes an extension of the bigravity framework that allows the mass of the graviton to become environmentally dependent by relying on the chameleon mechanism. It offers a novel way to reconcile massive gravity with local tests of General Relativity without invoking the Vainshtein mechanism. This shed a new light on the interplay between local and global tests of gravity, since deviation is expected on cosmological scales at early times. This extension gives more freedom in the parameter space, so that one can expect signatures on cosmological scales while Solar system constraints are satisfied. More generally, this demonstrates the importance of the screening mechanism in drawing the predictions.

In the context of bimetric theories, the stability of the homogeneous and isotropic universe requires (see footnote 1) the generalized Higuchi bound condition (7) to be satisfied. In the standard bigravity theory on a de Sitter background spacetime, it reduces to the classical Higuchi bound (31). In the standard bigravity theory,

unfortunately, the bound is violated in the early universe with sufficiently large H . In the chameleonic extension of bigravity proposed in this letter, on the contrary, m_T depends on the environment in such a way that m_T^2 scales with the matter energy density, $m_T^2 \propto \rho$. This significantly broadens the regime of applicability of the bimetric theory. Indeed, for de Sitter backgrounds the ratio m_T^2/H^2 does not depend on the vacuum energy density in the Jordan frame (see (29)) and thus the bound (30) is automatically satisfied at all scales once it is satisfied at one scale. For example, if the late-time universe dominated by dark energy is stable then the inflationary universe in the early universe is also stable for the same choice of theory parameters.

During the radiation-dominated era, one can find a scaling solution in which ξ and c ($\neq 1$) stay constant and yet each term of (13) scales as $1/a^4$. This scaling solution is an attractor of the system (13)-(17) under a certain condition and leads to the constancy of the ratio m_T^2/H^2 again. However, the cosmological evolution and its stability all the way from the early radiation-dominated epoch to the present epoch with acceleration remains to be studied [10].

In conclusion, the extension of bigravity models proposed in this letter is very generic and robust. Indeed we

have considered only a simple example. One can easily allow for different scalings for the β_i . Also, the coupling to the field may not be universal and, e.g. may be different for the dark matter sector, which would also help to evade local and cosmological constraints simultaneously. It is also possible to generalize the relation between the Einstein frame and the Jordan frame from the simple conformal transformation $\tilde{g}_{\mu\nu} = A^2(\phi)g_{\mu\nu}$ to more elaborated one such as a disformal transformation of the form $\tilde{g}_{\mu\nu} = A^2(\phi)g_{\mu\nu} + B(\phi)\partial_\mu\phi\partial_\nu\phi$. Our companion paper shall detail all these issues.

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- [1] C. de Rham, Living Rev. Relativ. **17**, 7 (2014); A. Schmidt-May and M. von Strauss, J. Phys. A **49**, no. 18, 183001 (2016).
 - [2] A.I. Vainshtein, Phys. Lett. B **39**, 393 (1972); C. Defayet, G.R. Dvali, G. Gabadadze, and A.I. Vainshtein, Phys. Rev. D **65**, 044026 (2002); E. Babichev and C. Defayet, Class. Quant. Grav. **30**, 184001 (2013).
 - [3] A. Higuchi, Nucl. Phys. B **282**, 397 (1987).
 - [4] J. Khoury, and A. Weltman, Phys. Rev. Lett. **93**, 171104 (2004), [astro-ph/0309300]; J. Khoury, and A. Weltman, Phys. Rev. D **69**, 044026 (2004), [astro-ph/0309411].
 - [5] A. De Felice, A. E. Gumrukcuoglu, S. Mukohyama, N. Tanahashi and T. Tanaka, JCAP **1406**, 037 (2014), [arXiv:1404.0008 [hep-th]].
 - [6] A. De Felice, T. Nakamura, and T. Tanaka Prog. Theor. Exp. Phys. **2014**, 043E1 (2014), [1304.3920].
 - [7] M. Fasiello and A. J. Tolley, JCAP **1312**, 002 (2013), [arXiv:1308.1647 [hep-th]].
 - [8] A. E. Gumrukcuoglu, S. Mukohyama and T. P. Sotiriou, Phys. Rev. D **94**, no. 6, 064001 (2016), [arXiv:1606.00618 [hep-th]].
 - [9] G. D’Amico, G. Gabadadze, L. Hui and D. Pirtskhalava, Phys. Rev. D **87**, 064037 (2013) doi:10.1103/PhysRevD.87.064037 [arXiv:1206.4253 [hep-th]].
 - [10] A. De Felice, S. Mukohyama, M. Oliosi, J.-P. Uzan, Y. Watanabe, in preparation.